**Graphical Analysis**

So far we have discussed uniform motion, or motion at a constant speed in a straight line. Only a few things move with uniform motion for long periods of time. Generally they speed up or slow down or change direction.

We can interpret information about the motion of an object simply by looking at the graph. You have already learned that:

**The slope of a distance versus time graph gives us the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of the object.**

Last day we learned the difference between average speed and average velocity. In terms of graphing

**The slope of a position versus time graph gives us the \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ of the object.**

Without using numbers let us consider the following position versus time graphs and ask ourselves the following questions:

1. What direction is the object moving?
2. Is the object stopped, moving with a constant speed, speeding up or slowing down?
3. Is the motion uniform or non-uniform?

**Recall**:

Positive directions include 🡪

Negative directions include 🡪

(A) (B)

t (s)

d (m)

t (s)

d (m)

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(C) (D)

t (s)

d (m)

t (s)

d (m)

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**Velocity-Time Graphs and Acceleration**

Position-time graphs are useful for representing the motion of object traveling at a constant velocity or changing from one constant velocity to another, but the motion of objects whose velocity is constantly changing is better represented by a velocity-time graph.

|  |  |
| --- | --- |
| **Time (s)** | **Velocity (m/s)** |
| 0 | 0 |
| 5 | 5 |
| 10 | 10 |
| 15 | 15 |
| 20 | 20 |
| 25 | 25 |
| 30 | 30 |

vt11-1

The graph above shows the changing velocity of a car. It is a straight line and it represents an increase in velocity. On closer examination, you will see that the car increases its velocity by 5.0 m/s during each five second interval. This is an acceleration of 1.0 m/s2. Since the acceleration for each interval is the same, the graph illustrates uniform acceleration. Whenever a velocity-time graph is a straight line, it represents uniform acceleration.

To determine the acceleration form a velocity versus time graph, you simply calculate its slope. That is

**The acceleration is equal to the slope of a velocity-time graph.**

Note: Uniform acceleration is NOT uniform motion.

**Positive, Negative, and Zero Acceleration**

Example 1: Examine the velocity-time graph below. It shows examples of three different types of acceleration. Calculate the acceleration for each segment.

|  |  |  |
| --- | --- | --- |
| A | B | C |
|  |  |  |

vt11-2

1. The object starts from rest (at zero velocity) and accelerates steadily for a while. The velocity is increasing. If acceleration and velocity have the same sign the object is speeding up.
2. After 10 s, it stops accelerating and then travels at a constant velocity for 10 s. The slope of this part of the graph is zero, that is, the acceleration is zero. IMPORTANT: the object is not stopped, it has a constant velocity.
3. Then the object slows down. In each second, it loses the same amount of velocity. The slope of the graph in this section is negative, making the acceleration negative. If acceleration and velocity have opposite signs, the object is slowing down.

If line segment C were extended so that it went below the axis, the acceleration would still be negative. However, the object would now be moving faster in a negative direction.

Note: Large negative accelerations are experienced when cars and passengers are stopped abruptly, assuming that the car’s original velocity was positive.

Example 2: Use the velocity-time graph below to determine the object’s acceleration for

(a) the first 10 s of its motion, and

(b) the time interval of 10 s to 15 s.

vt11-5

***Practice Problems***

1. Calculate the acceleration of the object below for the time interval 15 s to 20 s.

vt11-5

1. For each letter section on the graph below, calculate the acceleration of the object.

vt11-3

1. Describe the motion of the object for each time interval.

A \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

B \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

C \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

D \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

E \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

1. Construct a velocity-time graph for

(a) a car that starts from rest and accelerates at 8 m/s2 for 10 s.

(b) a runner who runs at a constant velocity of 8.0 m/s for 5.0 s, then slows down at a uniform rate and stops in 2.0 s.

1. The graph below shows the motion of a car accelerating from a stop at an intersection.

(a) How fast was the car moving at the following times: 2.0 s, 4.0 s, and 15.0 s?

(b) Determine the acceleration during the following time intervals: 0 to 4.0 s, 4.0 s to 10.0 s, and 10.0 s to 15.0 s.

vt11-4

**Displacement from a Velocity-Time Graph**

**The displacement in any interval is given by the area under a**

**velocity-time graph for that interval.**

Note: The area under a rectangular section is given by  and the area under a triangular section is given by .

Practice Problems

1. Find the displacement represented by each of the following velocity-time graphs:

(a)

vt13-1a

(b)

vt13-1b

(c)

vt13-1c

(d)

vt13-1d

(e)

vt13-1e

(f)

vt13-1f

(g)

vt13-1h

## **Using Velocity-Time Graphs to Calculate Distance or Displacement**

The area under or above the line can be found by dividing the shape of the graph below into 2 triangles and a rectangle. The area of the triangle can be found and then the area of the rectangle can be found. The total area under the graph above the x-axis and/or below the x-axis is considered either distance or displacement.

**NOTE:** Do not forget that the slope of a v-t graph means acceleration.

16

12

8

4

0

-4

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| **Velocity**  **m/s** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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2 4 6 8 10 12 14 16 18 20

**Time (s)**

**Distance**: A = 1/2bh + 1/2bh + lw

= ½(2s)(4m/s) + ½(4s)(8m/s) + (10s)(8m/s)

= 4 m + 16m + 80m

= 100 m

**Displacement:** A = **-**1/2bh + 1/2bh + lw

= **-**½(2s)(4m/s) + ½(4s)(8m/s) + (10s)(8m/s)

= **-**4m + 16m +80 m

= 92 m

**Graphing Review**

A. Position-time graphs and velocity-time graphs for uniform motion

* Slope of a position-time graph represents velocity
* For uniform motion, object moves in a straight line at constant speed, so velocity and speed have the same value or magnitude

|  |  |  |
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| mc73dvst | - object moves to right, north, up or east  - constant velocity |  |
| mc73dvst | - object moves to left, south, down or west  - constant velocity |  |
| mc73dvst | - object stopped |  |

B. Position-time graphs and Velocity-time graphs for accelerated motion (p. 390-393)

* Slope of a position-time graph represents velocity (tangent technique gives instantaneous velocity)
* Note: Imagine constructing a series of tangent lines to these curves and visualize whether they get steeper (speeding up) or flatter (slowing down) as time increases.
* Slope of velocity-time graph represents acceleration
* If both accel. & vel. are in the same direction (both are positive or both are negative), then the object is *speeding up*
* If both accel. & vel. are in opposite directions (one is positive and the other is negative) then the object is *slowing down*

|  |  |  |
| --- | --- | --- |
| mc73dvst | - object speeds up  - possible directions are to the right, north, up or east | mc67cvvst |
| mc73dvst | - object slows down  - possible directions are to the right, north, up or east | mc67avvst |
| mc73dvst | - object speeds up  - possible directions are to the left, south, down or west | mc67bvvst |
| mc73dvst | - object slows down  - possible directions are to the left, south, down or west | mc67dvvst |